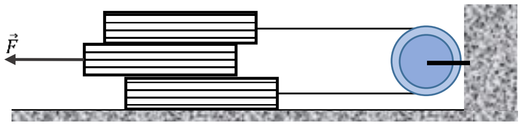
PHYS 101 – General Physics I Midterm Exam

Duration: 120 minutes Saturday, 18 November 2023; 09:30

1. Three identical books of mass m are stacked on top of each other on a frictionless horizontal surface. There is friction between the book surfaces for which the kinetic friction coefficient and static friction coefficient is the same and given as μ. The top and bottom books are connected to each other with a massless inextensible string over a massless and frictionless pulley as shown in the figure. Magnitude of the gravitational acceleration is g. A horizontal force of magnitude F is applied to the middle book as shown.

(a) (6 Pts.) Draw free-body diagrams for the books.

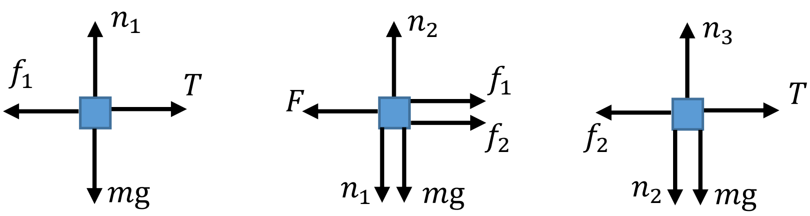
(b) (6 Pts.) What is the maximum value of F so that none of the books move?



(c) (6 Pts.) Suppose that the value of F is such that the top book slides over the middle book, but the lower two books move together. Find the acceleration of all books in such a case.

(d) (7 Pts.) What is the maximum value of F for part (c) to be true?

Solution: (a)



(b) Newton's second law for equilibrium in the horizontal direction is written as

$$f_1 = T\,, \qquad f_1 + f_2 = F, \qquad f_2 = T \;\rightarrow\; f_1 = f_2 = T = \frac{F}{2}.$$

For the top book to not slide on the middle book $f_1 \le \mu n_1 \rightarrow \; F/2 \le \mu mg \rightarrow \; F \le 2\mu mg$.

For the bottom book to not slide on the middle book $f_2 \le \mu n_2 \rightarrow \; F/2 \le 2\mu mg \rightarrow \; F \le 4\mu mg$.

So the condition for none of the books to move is $F \le 2\mu mg$.

(c) If the two lower books move together without sliding on top of each other, the magnitude of the acceleration of all three books will be the same. Newton's second law is written as

$$T - f_1 = ma\,, \qquad F - f_1 - f_2 = ma\,, \qquad f_2 - T = ma \;\rightarrow\; F - f_1 - T = 2ma \;\rightarrow\; F - 2f_1 = 3ma.$$

Since now the top book is sliding on the middle book, we have $f_1 = \mu n_1 = \mu mg$. Therefore

$$a = \frac{F - 2\mu mg}{3m}.$$

Note that $a > 0$ if $F > 2\mu mg$, as found in part (a).

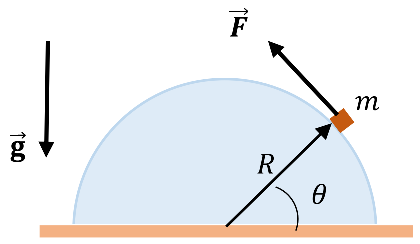
(d) Eliminating T and solving for f_2, we find

$$f_2 = T + ma = f_1 + 2ma \;\rightarrow\; f_2 = \frac{2}{3}F - \frac{1}{3}\mu mg\,.$$

For the middle book not to slide on the bottom book we need $f_2 \le \mu n_2 = \mu(n_1 + mg) = 2\mu mg$. So, we have

$$\frac{2}{3}F - \frac{1}{3}\mu mg < 2\mu mg \;\rightarrow \mathrm{F} < \frac{7}{2}\mu mg\,.$$

2. A small block of mass m is moved up along a fixed track that forms a semicircle of radius R in the vertical plane, as shown in the figure. The block moves very slowly (at equilibrium) with constant speed maintained by a varying tangential force along the track. The coefficient of friction between the block and the circular track is μ_k. Magnitude of the gravitational acceleration is g.



(a) (5 Pts.) Draw a free body diagram for the block when it is at the position shown in the figure.

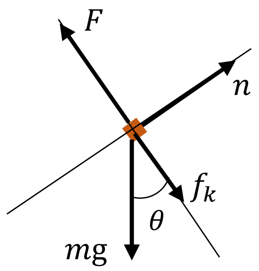
(b) (5 Pts.) Find the expression for the magnitude of the force $\vec{\boldsymbol{F}}$ as a function of the angle θ during the motion.

(c) (5 Pts.) Find the work done by the gravitational force as the block moves from $\theta = 0$ position to $\theta = \pi/2$ position.

(d) (5 Pts.) Find the work done by the friction force as the block moves from $\theta = 0$ position to $\theta = \pi/2$ position.

(e) (5 Pts.) Find the work done by the force $\vec{\boldsymbol{F}}$ as the block moves from $\theta = 0$ position to $\theta = \pi/2$ position.

Solution:(a)



(b) Since the block is moving kinetic friction is effective. Newton's second law is written as

$n = mg\sin\theta$, $\quad F - mg\cos\theta - f_k = 0$, $\quad f_k = \mu_k n = \mu_k mg\sin\theta$,

$F = mg(\cos\theta + \mu_k \sin\theta)$

(c) Gravitational force is conservative so $W_g = -\Delta U_g = -mgR$.

(d) By definition, work done by the friction force is $W_f = \int \vec{\boldsymbol{f}}_k \cdot d\vec{\mathbf{r}}$.

Along the circular path $|d\vec{\mathbf{r}}| = R\,d\theta$ with direction antiparallel to the direction of the force of friction. Therefore $\vec{\boldsymbol{f}}_k \cdot d\vec{\mathbf{r}} = -f_k R\,d\theta$, and since $f_k = \mu_k mg\sin\theta$, we have

$$W_f = -\int_0^{\pi/2} \mu_k mg\sin\theta\, Rd\theta = -\mu_k mgR\int_0^{\pi/2} \sin\theta\, d\theta = +\mu_k mgR\left(\cos\theta|_0^{\pi/2}\right) = -\mu_k mgR\,.$$

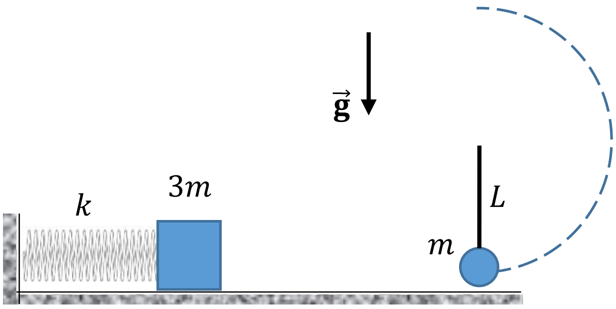
(e) Since $\Delta K = 0$, Using the work-energy principle, we find

$$W_{\text{tot}} = \Delta K \;\rightarrow W_g + W_f + W_F = 0 \;\rightarrow\; W_F = -\left(W_g + W_f\right) \rightarrow\; W_F = mgR(1 + \mu_k)\,.$$

3. Consider the figure, where we have a horizontal spring with stiffness constant k with one end fixed. A block of mass $3m$, which is free to slide on a frictionless horizontal surface, is pressed against the spring, compressing it a distance d from its equilibrium position. The block is not attached to the spring so, when released, starts moving and makes an **elastic** collision with a mass m hanging at the bottom end of a massless string of length L. Assume that the collision duration is very short so that motion during the collision is negligible.

(a) (10 Pts.) Find velocities of both masses after the collision in terms of k, m and d.

(b) (15 Pts.) What is the minimum value of d sufficient to cause the mass m to swing clear over the top of its arc?



Solution:

(a) The speed of the $3m$ block after it is released and seperates from the spring can be found using conservation of energy principle.

$$\frac{1}{2}kd^2 = \frac{1}{2}(3m)v_0^2 \;\rightarrow\; v_0 = d\sqrt{\frac{k}{3m}}.$$

Since the collision is elastic, bot linear momentum and kinetic energy is conserved. Let v_1 denote the velocity of the $3m$ mass and v_2 denote the velocity of the hanging mass after the collision. We have

$$3mv_0 = 3m\,v_1 + m\,v_2 \;\rightarrow\; , \qquad \text{and} \qquad \frac{1}{2}(3m)v_0^2 = \frac{1}{2}(3m)v_1^2 + \frac{1}{2}mv_2^2\,.$$

Solving the momentum equation for v_1, we find $v_1 = (v_0 - v_2/3)$. Using this result in the conservation of energy equation gives $v_2 = 3v_0/2$. So,

$$v_2 = \frac{3d}{2}\sqrt{\frac{k}{3m}} = \frac{d}{2}\sqrt{\frac{3k}{m}}\,, \qquad v_1 = v_0 - \frac{v_2}{3} = \frac{d}{2}\sqrt{\frac{k}{3m}}.$$

One gets the same result by using the relative velocity relation in elastic collisions, i.e., $v_0 = -(v_1 - v_2)$ instead of the energy conservation equation.

(b) In order for the mass m to swing clear over the top of its arc the string must not be slack. Since at the top

$$T + mg = mv_{2\mathrm{top}}^2/L,$$

and the minimum value of the tension T is zero, its speed at the top should satisfy $v_{2top} \geq \sqrt{Lg}$.

The total mechanical energy of the hanging mass is conserved after the collision. Therefore, we have

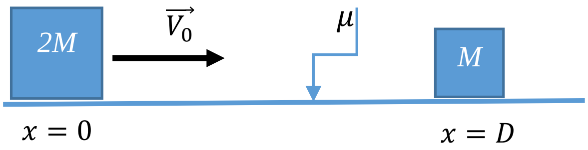
$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_{2top}^2 + mg(2L) \;\rightarrow\; v_{2top}^2 = v_2^2 - 4Lg \;\rightarrow\; v_2^2 - 4Lg \geq Lg \;\rightarrow\; v_2^2 \geq 5Lg.$$

Using the value of v_2 found, we get

$$\frac{3kd^2}{4m} \geq 5Lg \;\rightarrow\; d^2 \geq \frac{20Lmg}{3k} \;\rightarrow\; d_{\min} = \sqrt{\frac{20Lmg}{3k}}.$$

4. Two blocks of mass $2M$ and mass M are at $x = 0$ and $x = D$ on a horizontal surface. Both blocks have friction coefficient μ with the surface, where the difference between kinetic and static friction coefficient is negligible. Gravitational acceleration is given as g, and the blocks are small enough to be treated as point particles.

The mass 2M is given a horizontal speed V_0 towards M.



(a) (5 Pts.) What is the minimum V_0 so that the $2M$ block can hit the M block?

Assume that V_0 is large enough for the two blocks to collide. Assume further that the collision happens so fast that motion during the moment of collision is negligible. Find the final position of the $2M$ block if:

(b) (10 Pts.) the collision between them is completely inelastic (i.e. the blocks stick to each other);

(c) (10 Pts.) the collision is between them is elastic.

Solution:

(a) In order that the $2M$ block hits the other block its initial kinetic energy must be equal to or greater than the work done by the friction force over the distance D. This means

$$\frac{1}{2}(2M)V_0^2 \geq fD\,, \qquad f = \mu(2M)\mathrm{g} \;\to\; V_0 \geq \sqrt{2\mu \mathrm{g} D} \;\to\; V_{0\min} = \sqrt{2\mu \mathrm{g} D}\,.$$

(b) Let V_1 denote the velocity of the $2M$ block just before the collision. Work-energy principle implies

$$\Delta K = W_f \;\to\; \frac{1}{2}(2M)V_1^2 - \frac{1}{2}(2M)V_0^2 = -\mu(2M)\mathrm{g}D \;\to\; V_1 = \sqrt{V_0^2 - 2\mu \mathrm{g} D}\,.$$

Momentum is conserved in the collision. If we let V' denote the velocity of the stuck blocks just after the collision, we have

$$2MV_1 = 3MV' \;\to\; V' = \frac{2}{3}V_1 = \frac{2}{3}\sqrt{V_0^2 - 2\mu \mathrm{g} D}\,.$$

The stuck blocks will stop after moving a distance d when the work done by friction is equal to the kinetic energy just after the collision.

$$\frac{1}{2}(3M)V'^2 = \mu(3M)\mathrm{g}d \;\to\; \frac{2}{9}(V_0^2 - 2\mu \mathrm{g} D) = \mu \mathrm{g} d \;\to\; d = \frac{2}{9}\left(\frac{V_0^2}{\mu \mathrm{g}} - 2D\right)$$

Final position of both blocks is

$$D + d = \frac{2}{9}\frac{V_0^2}{\mu \mathrm{g}} + \frac{5}{9}D\,.$$

(c) Let $V_1{}'$ and $V_2{}'$ denote velocities of the $2M$ block and the M block just after the collision respectively. If the collision is elastic, we have $2MV_1 = 2MV_1' + MV_2' \;\to\; 2V_1' + V_2' = 2V_1$. Since the collision is elastic, we also have $V_2' - V_1' = V_1$. Solving these two equations, we get

$$V_1' = \frac{V_1}{3}\,, \qquad V_2' = \frac{4}{3}V_1.$$

Asd in the previous part, the $2M$ block will stop after moving a distance d' when the work done by friction is equal to the kinetic energy just after the collision.

$$\frac{1}{2}(2M)V_1'^2 = \mu(2M)\mathrm{g}d' \to \frac{1}{2}\left(\frac{V_1}{3}\right)^2 = \mu \mathrm{g} d' \to d' = \frac{1}{18}\left(\frac{V_0^2}{\mu \mathrm{g}} - 2D\right) \to D + d' = \frac{1}{18}\frac{V_0^2}{\mu \mathrm{g}} + \frac{8}{9}D\,.$$